

# Giant Negative Mobility of Janus Particles in a Corrugated Channel

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We numerically simulate the transport of elliptic Janus particles along narrow two-dimensional channels with reflecting walls. The self-propulsion velocity of the particle is oriented along either their major (prolate) or minor axis (oblate). In smooth channels, we observe long diffusion transients: ballistic for prolate particles and zero-diffusion for oblate particles. Placed in a rough channel, prolate particles tend to drift against an applied drive by tumbling over the wall protrusions; for appropriate aspect ratios, the modulus of their negative mobility grows exceedingly large (giant negative mobility). This suggests that a small external drive suffices to efficiently direct self-propulsion of rod-like Janus particles in rough channels.

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## I. INTRODUCTION

Self-propulsion [1] is the ability of most living organisms to move, in the absence of external drives, thanks to an “engine” of their own. Self-propulsion of fabricated micro- and nano-particles (artificial microswimmers)[2–5] is a topic of current interest in view of applications to nanotechnology. Their direct experimental observation became affordable with the synthesis of a new class of asymmetric microswimmers, which propel themselves by generating local gradients in the suspension environment (self-phoretic effects) [6]. Such particles, called Janus particles (JPs) [7, 8], consist of two distinct sides, only one of which is chemically or physically active. Thanks to their functional asymmetry, these active particles can induce either concentration gradients (self-diffusiophoresis) by catalyzing a chemical reaction on their active surface [9–12], or thermal gradients (self-thermophoresis), e.g., by inhomogeneous light absorption [13] or magnetic excitation [14].

Much effort is presently directed to achieve reliable transport control of JPs in confined geometries [15–17]. The ability of Janus microswimmers to perform directed autonomous motions through periodic arrays [12] and asymmetric channels [18] surely is a suggestive option. Such devices do operate in the absence of external drives or gradients, but at the price of strict fabrication requirements regarding their geometry. In this Letter we propose a more affordable option to direct the motion of JPs along a channel. Under appropriate conditions involving the geometry of both the particle and the channel, a tiny external drive (even in the absence of other biases) can orient the self-propulsion velocity of the microswimmers *against* the drive; a phenomenon known as absolute negative mobility (ANM) [19, 20]. The roughness of the channel walls, mimicked here by randomly inserting small

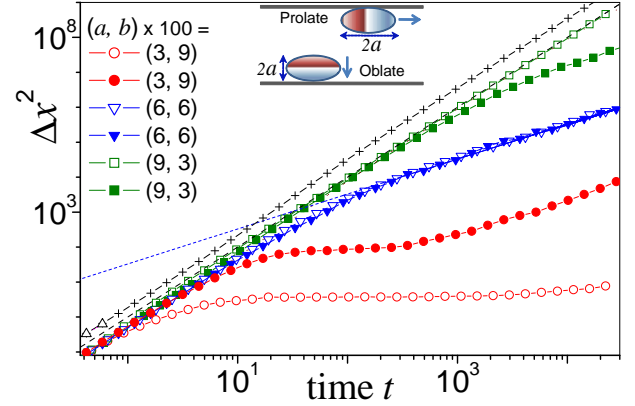


FIG. 1: (Color online) Diffusion of elliptical JPs with semi-axes  $a$  and  $b$  (see legend) in a smooth channel of width  $y_L = 1$ . Thermal noise is  $D_0 = 10^{-4}$  (empty symbols) and  $6.4 \times 10^{-3}$  (filled symbols); self-propulsion parameters are  $v_0 = 1$  and  $D_\theta = 0.03$ . The ballistic transient,  $\Delta x^2 = (v_0 t)^2$  (dashed line), and the normal diffusion law,  $\Delta x^2 = 2D_{\text{eff}} t$  (see text, dotted line) are drawn for a comparison. Example of persistent ballistic diffusion transient (crosses):  $a = 0.15$ ,  $b = 0.03$ ,  $y_L = 10$ ,  $v_0 = 2$ ,  $D_0 = 0.02$  and  $D_\theta = 0.003$  [12]. Sketch: sliding prolate (top) and stuck oblate disk (bottom). In both cases  $\mathbf{v}_0$  (single-pointed arrow) is oriented along the  $a$  axis.

transverse wall protrusions, can drastically enhance this phenomenon, thus producing a *giant* absolute negative mobility. These features suggest most sensitive control techniques on JP transport with beneficial applications to nanotechnology and medical sciences [2, 7]

## II. MODEL

Shape is known to play a central role in the diffusive dynamics of confined JPs [21, 22]. For this reason we consider two-dimensional (2D) channels and elongated particles, modelled as elliptical disks with semi-axes  $a$  and  $b$ . Actual rod-like JPs can be synthesized through a variety of well-established techniques [15, 21, 23]. An elongated JP gets a continuous push from the suspension fluid, which in the overdamped regime amounts to a self-propulsion velocity  $\mathbf{v}_0$  with constant modulus  $v_0$ . Additional dynamical effects, neglected in the present model, are briefly discussed at the end. We assume that  $\mathbf{v}_0$  acts along the  $a$  axis of the particle:  $\mathbf{v}_0$  is parallel to the particle major axis for  $a > b$  (prolate JP) and orthogonal to it for  $a < b$  (oblate JP). The self-propulsion direction varies randomly with time constant  $\tau_\theta$ ; accordingly, the microswimmer mean self-propulsion path is  $l_\theta = v_0\tau_\theta$ .

The bulk dynamics of such a JP obeys the Langevin equation [24]

$$\dot{\mathbf{r}} = \mathbf{v}_0(t) + \mathbf{F} + \xi_0(t), \quad (1)$$

where  $\mathbf{r} = (x, y)$  denotes the position of the particle center of mass,  $\mathbf{F} = (F, 0)$  represents a d.c. external bias parallel to the channel axis, and  $\xi_0(t) = (\xi_{0,x}(t), \xi_{0,y}(t))$  models a Gaussian thermal noise with  $\langle \xi_{0,i}(t) \rangle = 0$  and  $\langle \xi_{0,i}(t)\xi_{0,j}(0) \rangle = 2D_0\delta_{ij}\delta(t)$ , with  $i, j = x, y$ . The self-propulsion velocity  $\mathbf{v}_0 = v_0(\cos\theta, \sin\theta)$  is oriented at an angle  $\theta(t)$  with respect to the  $x$  axis, where  $\theta(t)$  is a Wiener process,  $\dot{\theta} = \xi_\theta(t)$ , with  $\langle \xi_\theta(t) \rangle = 0$  and  $\langle \xi_\theta(t)\xi_\theta(0) \rangle = 2D_\theta\delta(t)$ . As a change in  $\theta$  corresponds to a rotation of the swimmer,  $D_\theta$  is related to both  $\tau_\theta$ ,  $D_\theta = 2/\tau_\theta$  [18, 24], and the average thermal diffusivity  $D_0$ ,  $D_\theta \propto D_0/ab$  [25]. To make contact with realistic experimental conditions in our simulations we chose parameters experimentally accessible. For instance, expressing times in seconds and lengths in microns, typical values for spherical JP of radius 0.5 – 2.0 are  $D_0 = 0.02 - 0.03$ ,  $D_\theta = 0.01$ , and  $v_0 = 0.05 - 0.5$  [12]

When confined to a cavity of size smaller than its self-propulsion length,  $l_\theta$ , the microswimmer undergoes multiple collisions with the walls and the confining geometry comes into play (Knudsen diffusion [26]). The Langevin equations (1) have been numerically integrated under the assumption that the channel walls were perfectly reflecting and the particle-wall collisions elastic [27].

## III. DIFFUSION TRANSIENTS

Transport of elongated JPs along a smooth channel is characterized by unit mobility,  $\mu = 1$ , where  $\mu \equiv v/F$  and  $v(F) = \lim_{t \rightarrow \infty} \langle x(t) \rangle / t$ . The particle shape becomes distinguishable when one looks at the dispersion,  $\Delta x^2(t) = \langle x^2(t) \rangle - \langle x(t) \rangle^2$ . Both prolate and oblate disks undergo surprisingly long transients, see Fig. 1, before the expected normal diffusion regime sets on. On

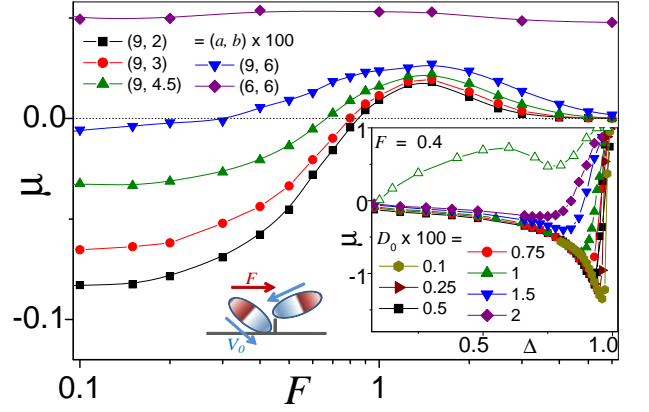


FIG. 2: (Color online) Mobility  $\mu(F)$  of a prolate JP driven along a septate channel for different values of its semi-axes  $a$  and  $b$  (see legend). Compartment parameters:  $x_L = y_L = 1$  and  $\Delta = 0.16$ ; self-propulsion parameters:  $v_0 = 1$  and  $D_\theta = D_0 = 0.03$ . Inset:  $\mu$  vs  $\Delta$  for  $F = 0.4$ ,  $a = 0.09$ ,  $b = 0.03$  and different  $D_0$  (solid symbols). The curve (empty triangles) for the oblate JP with  $a = 0.03$ ,  $b = 0.09$  at  $D_0 = 0.01$  is plotted for comparison. All the remaining parameters are as in the main panel. Sketch: a prolate JP tumbling over a winglet of the channel wall under the action of the drive (see text).

the contrary, circular disks seem to diffuse like ordinary point-like JPs [18, 24]: for  $t \gtrsim \tau_\theta$ ,  $\Delta x^2(t) = 2D_{\text{eff}}t$  with  $D_{\text{eff}} = D_0 + v_0l_\theta/4$ . Due to their collision against the walls, active prolate swimmers tend to slide parallel to the channel axis, which explains their ballistic diffusion transient with  $\Delta x^2(t) \sim (v_0t)^2$ , whereas oblate swimmers tend to pile up against the walls, thus suppressing longitudinal diffusion. Upon increasing the strength of the thermal noise,  $D_0$ , both transients are shortened, though at a different rate. This property can be explained by noticing that the onset of normal diffusion requires that the microswimmers are capable of inverting the direction of  $\mathbf{v}_0$  against the confining action of the channel walls. On the other hand, a U-turn can only occur when noise kicks the elongated particle out of its steady (sliding or stuck) state. This amounts to a noise-activated process with time constant of the order of [27]  $\tau_U \simeq \tau_0 \exp(|a - b|v_0/D_0)$ , where  $\tau_0 \sim \sqrt{v_0a^2/b}$ . Accordingly, for the  $a$  and  $b$  values of Fig. 1, transients are longer for the prolate than for the oblate particle. When  $\tau_U$  becomes of the order of or smaller than the crossing time  $y_L/v_0$ , boundary effects vanish and (bulk) normal diffusion is fully restored, i.e., transients become negligible. For strongly prolate, say, rod-like swimmers, ballistic transients may extend over exceedingly long time intervals even at room temperature (see example in Fig. 1). For this reason determining how dispersion scales with time can prove a delicate experimental issue [2].

#### IV. ABSOLUTE NEGATIVE MOBILITY

We consider now the case of a periodically-compartmentalized channel obtained by inserting equally-spaced transverse dividers, each bearing a small opening, or pore, centered on the channel axis. Let  $x_L$  and  $y_L$  be the longitudinal and transverse dimensions of the compartments and  $\Delta$  the pore size (see sketch in Fig. 3). Such a channel is termed septate [28–30] to stress the role of the compartment walls when compared with the smooth cross-section modulation of the so-called entropic channels [31]. Here, transport of elongated swimmers is governed by the pore crossing dynamics, which makes the mobility shape-dependent.

In Figs. 2 and 3 we plot the mobility of prolate JPs as a function of the drive, for different values of the aspect ratio  $a/b$  and the orientation constant  $D_\theta$ . The most prominent feature is the appearance of ANM branches at low  $F$ , which means that, under certain conditions, active swimmers may drift against the applied drive [19]. This effect (i) grows more apparent for large aspect ratios,  $a/b$  (Fig. 2); (ii) is limited to the domain  $F < v_0$ ; (iii) does not occur for oblate particles (an example was added to the inset of Fig. 2); and (iv) exhibits a resonant dependence on  $\tau_\theta = 2/D_\theta$  (Fig. 3, inset). For larger  $F$  the mobility turns positive, reaches a maximum, and finally decays to zero faster than  $1/F$  (*negative differential mobility* branch).

We now qualitatively interpret these properties by having recourse to the analytical arguments of Refs. [30]. First of all, the ANM mechanism can be explained by recalling that a driven prolate particle gets pressed longitudinally against the side walls of the channel compartments. As in Figs. 2 and 3,  $\Delta < 2a$ , the particle can slip through the wall openings only by rotating with  $\mathbf{v}_0$  (almost) orthogonal to the channel cross-section [27]. Again, this is a noise-activated mechanism with Arrhenius factor  $\exp(-|a - b|(F \pm v_0)/D_0)$ , the sign  $\pm$  denoting the parallel or anti-parallel orientation of  $\mathbf{v}_0$  with respect to  $\mathbf{F}$ . Therefore, pore crossings are more likely to take place on the left side of the compartment than on its right side; hence the observed negative mobility. This argument also leads to the conclusion that ANM is ruled out for prolate JPs with  $F \geq v_0$  and oblate JPs at any drive. This standstill at zero current crossing with  $\mu(F) = 0$  for a prolate microswimmers shifts to lower  $F$  on increasing  $D_0$ , as dispersion induced by thermal fluctuations tends to hamper the particle drift opposite to the drive. An oblate microswimmer, oriented with its major axis orthogonal to the compartment wall tends to propel itself parallel to (rather than against) it, whereas the drive pulls it back to the right (see sketch in Fig. 3). This state of affairs cannot be overturned by thermal noise. Thus, for an oblate JP pore-crossings to the right are favored and the mobility is always positive.

For large drives,  $F \gg v_0$ , the particle mobility is suppressed. This is a typical effect characteristic of septate channels [29], which is not observed in smoothly-

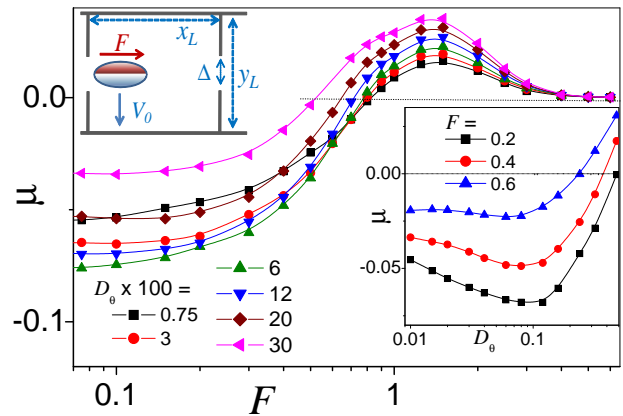


FIG. 3: (Color online) Mobility  $\mu(F)$  of a prolate JP driven along a septate channel for different values of  $D_\theta$  (i.e., the reciprocal of  $\tau_\theta$ , see legend). Compartment parameters:  $x_L = y_L = 1$  and  $\Delta = 0.16$ ; other simulation parameters:  $v_0 = 1$ ,  $a = 0.09$ ,  $b = 0.03$  and  $D_0 = 0.03$ . Inset:  $\mu$  vs.  $D_\theta$  for different  $F$ . All remaining parameters are as in the main panel. Sketch: oblate JP aligned as to leave a septate compartment to the left, being pulled back by  $F$ .

corrugated channels [31]. The particle leaves the compartment to the right, thus advancing a length  $x_L$ , only after an activation time,  $\tau_\perp \propto \exp(|a - b|(F + v_0)/D_0)$ , it needs to line up with the pore axis. Accordingly,  $\mu \sim x_L/(2\tau_\perp F)$ , in fairly good agreement with our data. Finally, the inset of Fig. 3 clearly indicates that the ANM is maximal for an optimal choice of  $\tau_\theta = 2/D_\theta$  (or  $l_\theta$ ). Note that for  $\tau_\theta \rightarrow 0$  the active random motion of a JP boils down to a standard Brownian motion with Einstein bulk diffusivity  $D_{\text{eff}}$ , and, therefore, its mobility must be positive. On the other hand, an elongated particle slips through a pore when, after turning perpendicular to the compartment wall, it diffuses toward the pore in a time,  $\tau_\parallel \sim y_L^2/2D_0$ , no longer than its orientation time-constant  $\tau_\theta$ . Based on our data, an optimal ANM at constant  $F$ , with  $F < v_0$ , is thus achieved by setting  $\tau_\parallel \sim \tau_\theta$ .

We mention in passing that we detected ANM also by driving prolate JPs along sinusoidally-corrugated channels (not shown). Contrary to what happens in septate channels, negative mobility sets on for more elongated particles, i.e., at higher  $a/b$ , and  $\mu$  approaches unity in the limit of large drives,  $F \gg v_0$ .

#### V. GIANT ABSOLUTE NEGATIVE MOBILITY

The magnitude of the backward rectification flow also depends on the compartment geometry and, in particular, on the pore size  $\Delta$ . In the inset of Fig. 2 we plotted the mobility of a prolate JP versus  $\Delta$  for  $F < v_0$  and different noise levels  $D_0$ . The modulus of the negative mobility  $|\mu|$  grows with  $\Delta$  up to an optimal value  $\Delta_M$ , which decreases with raising  $D_0$ ; for  $\Delta > \Delta_M$  the mo-

bility jumps abruptly to its bulk value,  $\mu = 1$ . This behavior is surprising as the ANM mechanism relies on the blocking action of the compartment walls [19, 20]. In the case of an elongated particle such an action is strongest when  $\mathbf{F}$  and  $\mathbf{v}_0$  point in the same direction [27], which causes a rectification flow against  $\mathbf{F}$ , i.e., negative mobility. However, as the pore size grows larger than the particle length,  $\Delta \gtrsim 2a$ , one would expect ANM to vanish. *Our data prove exactly the opposite.* More remarkably, on increasing  $\Delta$ ,  $|\mu|$  overshoots to its maximum value consistent with the ANM mechanism, that is,  $|\mu_M^{\text{ANM}}| = (v_0 - F)/(2F)$ . This estimate of  $|\mu_M^{\text{ANM}}|$  refers to the ideal optimistic case when a noiseless driven particle exits unhindered the channel compartments to the left with speed  $v_0 - F$  for half of the time, whereas during the rest of the time it sits stuck against the right compartment walls. Note that spontaneous-direction reversals occur on the time scale  $\tau_\theta$ . Most remarkably, the low-noise curves  $\mu(\Delta)$  in the inset of Fig. 2 exhibit sharp minima with  $|\mu_M| = \mu(\Delta_M)$  about twice the ANM estimate  $|\mu_M^{\text{ANM}}|$ .

A simple explanation of this finding is illustrated by the sketch inserted also in Fig. 2. The large values of  $|\mu_M|$  we observed imply that the *self-propulsion velocity of the driven JP (almost) always points against  $\mathbf{F}$* . For a particle confined to a narrow channel with  $l_\theta \gg y_L$ , this is certainly true for half the time. For the remaining half of the time, however,  $\mathbf{v}_0$  would spontaneously orient itself parallel to  $\mathbf{F}$ . This means that the collisions with the walls (and not the fluctuations of  $\theta(t)$ ) must be responsible for the quick rotation of  $\mathbf{v}_0$  to the opposite direction. For large  $\Delta$  the compartment walls shrink to a pair of perpendicular winglets of length  $\delta = (y_L - \Delta)/2$ , sticking out of the channel walls. An elongated microswimmer hitting one such obstacle from the left, tumbles over it under the action of the torque exerted by the drive.

Of course, such a tumbling mechanism is most effective under a few simple conditions: (i) the tumbling time,  $\tau_\delta \sim \pi I/Fa$ , with  $I = (a^2 + b^2)/4$  denoting the moment of inertia of an elliptical disk of unit mass, has to be much shorter than  $\tau_\theta$ ; (ii)  $\delta$  has to be the shortest possible, to minimize the blocking action of the compartment walls; (iii) the noise level has to be lowered as much as possible, lest the particle diffuses past the obstacle without tumbling. An optimal choice for  $\delta$  seems  $\delta_M \sim b/2$ , which corresponds to  $\Delta_M \sim y_L - b$  (see Fig. 2, inset). For shorter  $\delta$  the particle can slide over the obstacle without rotating. Under these conditions  $\mu_M \simeq -(v_0 - F)/F$  and its modulus can grow very large, indeed! We propose to term this phenomenon *giant negative mobility* (GNM).

GNM is sustained by the self-propulsion of the driven microswimmers themselves, the drive just helping orient their velocities opposite to it. In such a scheme, contrary to ANM [19], the obstacles fabricated along the channel walls do not act as traps, but rather as centers of rotation (tumbling). To better assess the robustness of GNM, in Fig. 4 we plotted  $\mu$  versus  $\delta$  for both periodically and randomly distributed winglets, all of the same length

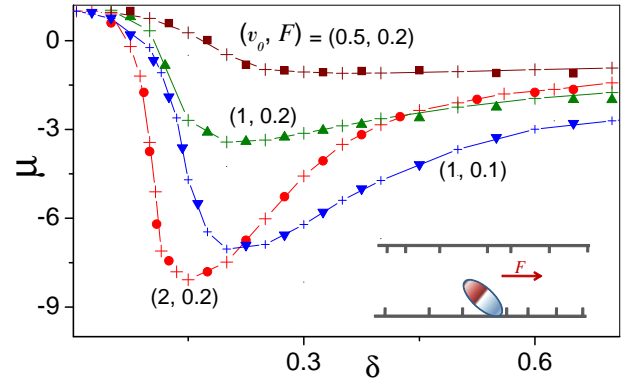


FIG. 4: (Color online) Giant negative mobility for a prolate JP driven along a rough channel of width  $y_L = 10$  with different  $v_0$  and  $F$  (see legend). On both sides winglets of length  $\delta$  are separated by a random distance  $l_n$ , uniformly distributed in the interval  $[5, 15]$  (solid symbols, see text); for a comparison we also simulated the corresponding case of equally-spaced winglets with  $l_n = 10$  (crosses). Other simulation parameters are:  $a = 1$ ,  $b = 0.25$ ,  $D_0 = 0.02$ , and  $D_\theta = 6 \times 10^{-3}$ .

$\delta$ . Randomized winglet distributions have been generated by scrambling regular distributions with period  $x_L$ , that is, by spacing the winglets a distance  $l_n$  apart, with  $l_n = x_{n+1} - x_n = x_L(1 + \delta_n)$ ,  $x_n$  being the position of the  $n$ -th winglet and  $\delta_n$  a random number uniformly distributed in the interval  $[-0.5, 0.5]$ . To closer mimic wall roughness, the winglet distributions on the upper and lower channel walls were independently generated. We selected particle and compartment sizes consistent with the experimental setups described in the literature [12]. For both winglet distributions our estimates for  $\Delta_M$  and  $\mu_M$  work quite well, meaning that GNM ought to be experimentally detectable at room temperature.

For all curves displayed in Fig. 4,  $|\mu_M|$  comes quite close to its upper bound  $2|\mu_M^{\text{ANM}}|$ . As anticipated, on lowering the self-propulsion diffusivity,  $v_0^2/2D_\theta$ , against  $D_0$ , the optimal winglet length  $\delta_M$  shifts to higher values and  $|\mu_M|$  slowly diminishes. Moreover, since particle tumbling is a local mechanism, GNM is rather insensitive to the actual distribution of the channel winglets. Analogously, winglets can be replaced by protrusions of different geometry to better model the roughness of channel walls. As long as they are sharp enough to engage the tips of rod-like JPs, the GNM mechanism does work (possibly facilitated by the hydrodynamic attraction between microswimmers and channel walls); if not, the smooth channel situation discussed at the very beginning would be recovered.

To emphasize the mechanisms responsible for the negative mobility of elongated JP's, in Eq. (1) we neglected a number of additional effects which may prove experimentally appreciable. We started assuming that self-propulsion is fully described by the random velocity  $\mathbf{v}_0$ , thus implying the absence of chiral (or torque) and inertial (or viscous) terms [32]. We also adopted scalar trans-



lational,  $D_0$ , and orientational diffusivity,  $D_c$ , despite the elongated shape of the active swimmer. A more detailed modeling of the active roto-translational dynamics [25] would have no substantial impact on the occurrence of negative mobility; the corresponding hydrodynamic corrections to the particle dynamics can be accounted by an appropriate rescaling of the model parameters [33]. More importantly, we ignored hydrodynamic effects, which not only favor clustering in dense mixtures of JP's [34, 35], but may even cause their capture by the channel walls [36]. Therefore, in the case of single diffusing JP's, hydrodynamic effects can only reinforce the flow of particles at the boundary, thus enhancing the predicted diffusion transients and the tumbling mechanism responsible for GNM. On the contrary, hydrodynamic effects are known to impact on the selective translocation of elongated JP's through narrow pores [37],  $\Delta \gg y_L$ , and, therefore, are likely to hamper the direct observation of ANM.

## VI. CONCLUSIONS

We numerically simulated the transport of elongated Janus particles driven along a narrow channel. Key transport quantifiers, like mobility and diffusivity, strongly depend on the particle shape. Diffusion in smooth channels is characterized by exceedingly long transients, either ballistic or non-diffusive, respectively, for prolate and oblate active microswimmers. In compartmentalized channels with narrow pores prolate Janus particles undergo absolute negative mobility, as an effect of the translational symmetry breaking due to the drive. More importantly, when the compartment di-

viders shrink to small side winglets, possibly randomly distributed along the channel walls, rod-like active particles greatly enhance their negative mobility, as the combined action of drive and channel roughness systematically reorients the particle self-propulsion velocity opposite to the drive itself. As the geometric and dynamical parameters used in our simulations closely compare with those reported for actual experimental set-ups, we are confident that giant negative mobility can soon be demonstrated, thus allowing a more effective transport control of active microswimmers. Such a control technique can be exploited, e.g., for medical applications, such as drug delivery via JPs to physiological opposing regions. Moreover, a dilute binary mixture of JP's of different shape can be driven along one such stylized rough channel so as to generate a two-way traffic (i.e., with mobilities of opposite sign), without having recourse to a parallel two-channel architecture with opposite drives..

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